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Using Stochastic Mortality Models to Measure Longevity Risk in Developed Countries

Chou-Wen Wang
Department of Risk Management and Insurance
University of National Kaohsiung First University of Science and Technology
2 Jhuoyue Rd., Nanzih, Kaohsiung City, 811, Taiwan

Yu-Ling Liou
Corresponding Author, Department of Risk Management and Insurance
University of National Kaohsiung First University of Science and Technology
2 Jhuoyue Rd., Nanzih, Kaohsiung City, 811, Taiwan
E-mail: u9527901@ccms.nkfust.edu.tw
Tel: 886-7-6011000; Fax: 886-7-6011069

Chin-Wen Wu
Department of Finance and Institute of Financial Management Nanhua University
No. 55, Sec. 1, Nanhua Rd., Zhongkeng, Dalin Township
Chiayi County 62248, Taiwan, R.O.C

Abstract

This study presents a stochastic mortality model, which is an extension of the Cairns-Blake-Dowd model (CBD) (Cairns and David et.al, 2009), to include an age component to account for distinct age effects among various age groups. The proposed extension is evaluated against well-known mortality models, including the Lee-Carter model (Lee and Carter, 1992), the Renshaw and Haberman model (Renshaw and Haberman, 2006) and the CBD model (Cairns and Blake, 2006b) and its variants (Cairns and David et.al, 2009). To model longevity risk, this study used data from Japan, the Asian nation with the highest life expectancy and UK, the highest life expectancy rate in Europe. A variety of measures were used to compare the models; the Bayesian information criterion was used to measure the parsimony of the models, and parameter estimation was used to evaluate the robustness of the models between different fitting periods. The results show that our proposed model is a promising approach in mortality modeling, producing empirical results that can greatly inform the insurance industry and governmental policies.

Keywords: Longevity Risk, Lee-Carter Model, Renshaw and Haberman Model, Cairns-Blake-Dowd Model

1. Introduction

In recent decades, emerging longevity risk issues have received increasing attention from academic scholars, the insurance industry and governments. Longevity risk results from the underestimation of human life expectancy in the preliminary plans of insurance companies and governments. Financial products such as life insurance, pension plans and governmental policies can experience serious losses

as a result of longevity risk. More accurate mortality models with improved mortality rate projections can provide a solution to this problem and can allow insurance companies and governments to redesign their financial products, strategies and policies. Although alternative approaches may also address longevity bonds (Chen and Cummins, 2010) or securitization (Cox and Lin et. al.,2010; Wills and Sherris, 2010) this study focused on the development of alternative mortality models.

Cairns and Blake (Cairns and Blake, 2006b) classified mortality models into two categories: the continuous-time framework (e.g., Milevsky David Promislow, 2001; Dahl, 2004; Biffis, 2005; Schrager, 2006; Dahl and Moller, 2006) and the discrete-time framework (e.g., Lee and Cater, 1992; Cairns and Blake, 2006b; Renshaw and Haberman, 2006; Cairns and David et. al., 2009). The well-known Lee-Carter (LC) model (Lee and Cater, 1992) is a discrete-time mortality model with a parsimonious structure and generally good performance. However, the LC model has several disadvantages, and many modifications have been proposed to address these issues. Renshaw and Haberman (Rensahaw amd Haberman, 2006) proposed a modification (called RH in this article) that added a cohort effect to the LC model, which improved performance when a cohort effect exists in the mortality data. However, Cairns and Blake (Cairns and Blake et. al., 2008) argued that the RH model suffers from a lack of robustness. Along with Dowd, they proposed a two-factor model (Cairns and Blake, 2006b) (called CBD) to model longevity risk. This model focuses on older populations and produces excellent results. Subsequently, Cairns et al. (Cairns and David et. al., 2009) proposed several modifications of CBD and used rigorous model selection criteria to evaluate the models with data from England, Wales and the United States. Their results showed that the CBD modification (called CBD2 in this article) exhibited a good balance of goodness of fit and robustness. Furthermore, they argued that no single best model exists, suggesting that users choose a model according to the circumstances and consider the problems of data-dominated models.

In this paper, we propose a modification to the CBD model called CBDE. This modification adds an age-specific vector to capture variance among different age groups. Mortality data from two developed countries were used: Japan, a developed country in Asia with a large elderly population, and the UK, a developed country in Europe. Cairns et al. (Cairns and David et. al., 2009) used an explicit model comparison, and we used the model evaluation process from their study to measure the goodness of fit and the robustness of the LC, RH, CBD, modified CBD and CBDE models with mortality data from Japan and the UK. According to BIC and parameter robustness, the CBDE model is a robust model for mortality data of Japan and the UK.

The remainder of this article is organized as follows: section 2 reviews the existing mortality models, section 3 describes our experimental design, section 4 illustrates the expected experimental results, and the final section summarizes the study's contributions.

2. Mortality Models

In this section, we briefly review seven popular mortality models and discuss their advantages and limitations. These models include the Lee-Carter model (Lee and Carter, 1992) and its extension, Renshaw and Haberman model (Renshaw and Haberman, 2006) and the CBD model and its extensions (Cairns and David et. al., 2009).

2.1. Lee-Carter Model (LC; LC Model) and Its Extension

Lee and Carter (Lee and Carter, 1992) proposed a mortality model for the central death rate series $\mu_{x,t}$ that reduces the dimensionality of the problem by identifying a single time index and summarizing historical trends. The structure of the LC mortality model is as follows:

$$\ln m_{x,t} = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t} \tag{2.1}$$

where the parameter α_x describes the average age-specific empirical mortality rates, κ_t represents the general mortality level, and the decline in mortality at age *x* is expressed by β_x. The term $\varepsilon_{x,t}$ indicates the deviation of the model from the observed log-central death rates and is assumed to be white noise with a mean of zero and relatively small variance (Lee, 2000). Specifically, $\varepsilon_{x,t}$ denotes an independent and identically distributed $N\left(0,\sigma^2\right)$ random variable, depicting an age- and time-specific effect not captured by the model. In a unique solution, Lee and Carter (Lee and Carter, 1992) employed two constraints: the sum of β_x coefficients equal to 1 and the sum of κ_t coefficients equal to 0, namely, $\sum_x \beta_x = 1$ and $\sum_t \kappa_t = 0$.

A variety of disadvantages arise in connection with the Lee-Carter model. An improvement is to incorporate a cohort effect to the Lee-Carter mode. Consequently, Renshaw and Haberman (Renshaw and Haberman, 2006) proposed an age-period-cohort mortality models (RH; RH model):

$$\ln m_{x,t} = \alpha_x + \beta_x^{(1)}\kappa_t + \beta_x^{(2)}\gamma_{t-x} \tag{2.2}$$

where γ_{t-x} is a random cohort effect that is a function of the year of birth (*t* – *x*). The LC model is a special case of the RH model when $\beta_x^{(2)}\gamma_{t-x}$ equals zero. The RH model offers a significantly better fit to historical data if a cohort effect was found in the past.

2.2. CBD Model and Its Extension

Cairns, Blake, and Dowd (Cairns and Blake et. al., 2006b) proposed a two-factor model that models initial mortality rates rather than a central mortality rate; this approach attempts to capture the dynamics of older age groups. Cairns and David et al. (Cairns and David et. al., 2009) then proposed several modifications of CBD. One generalization of the CBD model is of the form (CBD2; CBD2 model):

$$Logit\left(q(t,x)\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x}) + \kappa_t^{(3)}\left((x-\overline{x})^2 - \hat{\sigma}_x^2\right) + \gamma_{t-x} \tag{2.3}$$

where *q*(*t*,*x*) is the mortality rate for an individual of age *x* in year *t*; $\overline{x}$ is the mean age in the sample age range; $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ are summed to be a two-dimensional random walk with drift; $\kappa_t^{(3)}$ is a sensitivity to quadratic term to the age effect inspired by the possible curvature identified in the $Logit\left(q(t,x)\right)$ plots in the U.S. data; the constant $\hat{\sigma}_x^2 = n_a^{-1}\sum_t (x-\overline{x})^2$ is the mean of $(x-\overline{x})^2$; and γ_{t-x} is the cohort effect and satisfies the constraint $\sum_{c\in C}\gamma_c = 0$ and $\sum_{c\in C} c\gamma_c = 0$, where *c* is the set of cohort years of birth that have been included in the analysis. Note that we obtain the CBD0 model of Cairns, Blake, and Dowd (Cairns and Blake et. al., 2006b) when $\kappa_t^{(3)} = \gamma_{t-x} = 0$ and the CBD1 model when $\kappa_t^{(3)} = 0$. The CBD model has been widely adopted to investigate hedging and the securitization of longevity risk (Cairns and Blake et. al., 2006a). Data from England, Wales, and the United States have been used to demonstrate that the inclusion of a cohort effect can lead to a better fit (Cairns, 2007).

The second generalization of the CBD model, CBD3 model, builds on experience with fit in the RH model and satisfies

$$Logit\left(q(t,x)\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x}) + \gamma_{t-x}(x_c - x), \tag{2.4}$$

where x_c is constant parameter to be estimated. This model suggests that the impact of the cohort effect, γ_c, for any specific cohort, diminishes over time (i.e., γ_c decreases with age) instead of remaining constant. Models CBD1 to CBD3 are each an extension of the original CBD0 model and include some allowance for the cohort effect. Consequently, models CBD1 to CBD3 can be described as members of the family of generalized CBD models.

3. The Extension of the CBD Model (CBDE; CBDE Model)

In the CBD model, the mortality index $\kappa_t^{(2)}$ is only related to $(x-\bar{x})$ which affects mortality-rate dynamics at higher ages much more than at lower age. In the LC and RH models, however, the mortality index $\kappa_t^{(2)}$ is also related to the age-specific coefficient β_x. We therefore extended the LC and CBD2 models to create the following model:

$$Logit\left(q(t,x)\right)=\kappa_t^{(1)}+\kappa_t^{(2)}\beta_x(x-\bar{x})+\kappa_t^{(3)}(x-\bar{x})^2+\gamma_{t-x}. \tag{2.5}$$

This model contains five sets of parameters： $\kappa_t^{(1)}$, $\kappa_t^{(2)}$, $\kappa_t^{(3)}$, β_x and γ_{t-x}. Similar to the CBD2 model, $q(t,x)$ is also determined by the state variables $\left[\kappa_t^{(1)},\kappa_t^{(2)},\kappa_t^{(3)}\right]'$ which follow a three-dimensional random walk with drift. First factor, $\kappa_t^{(1)}$, interprets changes in mortality for all ages. Factor $\kappa_t^{(2)}$ explains how the variation between ages, which reflects the observed mortality rate, may differ among age groups over time. To identify curvature in the older age group, factor $\kappa_t^{(3)}$ and term $(x-\bar{x})^2$ provide variations that differ from $\kappa_t^{(1)}$. The last factor, γ_{t-x}, is the cohort component in this model. In contrast to CBD2, this model removed the term $\hat{\sigma}_x^2$.

To better understand the age-specific mortality rates with historical observations, we used a new component, β_x. This factor is similar to the Lee-Carter model and denotes the general shape of the logarithmic transformation of the age-specific mortality rates with historical observations. Furthermore, we adopted the same constants from CBD2 to obtain a unique solution that involves $\sum_{c\in C}\gamma_c=0$ and $\sum_{c\in C}c^2\gamma_c=0$. All of the parameters were estimated by the maximum likelihood method.

4. Experiments and Results

4.1. Data Set Description

To focus on longevity risk and analyze the model's performance, we examined data from the countries with the highest life expectancy in Asia and Europe. Japan and the UK were selected for our data set. According to statistics from the United Nations[1], Japan has the highest life expectancy in Asia. Empirical data from the UK were chosen because the UK has the highest life expectancy rate in Europe. All of the available data sets were obtained from the public Human Mortality Database (HMD)[2]. As shown in Table 1, data were available from a number of time periods. However, to ensure consistency with the time periods used by Cairns et al. (Cairns and David et. al., 2009), we used data from 1968 to 2008. All of the experiments were conducted using the statistics computing language R (R Development Core Team, 2010) and its packages (the LifeMetrics Toolkit released by J.P. Morgan (Coughlan and Epstein et. al., 2007)).

The mortality improvement trends for each country are shown in Figure and Figure 2; these figures cover the years 1950 to 2008, and the data are separated into ten-year periods (i.e., 1959, 1969, 1979, 1989, 1999 and 2008). The trends from the UK show fairly stable and linear improvement before the age of 80-85 years; after age 82.5, the trends show slight variability. In contrast, the trends from the Japan show the greatest volatility between the ages of 79-95 years. These results show that mortality trends are fairly flat and stable over time in UK.

[1] United Nations World Population Prospects: 2008 revision – http://esa.un.org/unpp/
[2] http://www.mortality.org/

Table 1: Available periods of Japanese and UK mortality data from the HMD

Region	Country	Available periods	Fitting Periods
Asia	Japan	1947-2008	1968-2008
Europe	UK	1922-2009	1968-2008

4.1. Model Selection Criteria

In this paper, we compare the seven models, LC, RH, CBD0, CBD1, CBD2, CBD3 and CBDE models according to the Bayesian information criterion (BIC), a general criterion for model selection among a class of parametric models with different numbers of parameters. When estimating model parameters using the maximum likelihood estimation, it is possible to increase the likelihood by adding parameters, which may result in over-fitting. The BIC resolves this problem by introducing a penalty term for the number of parameters in the model. The BIC for model *r* is defined as

$$BIC_\gamma = l(\hat{\phi}_\gamma) - \frac{1}{2}\nu_\gamma \log N \tag{2.6}$$

We conducted formal model comparisons based on these countries. For each model, we estimated the $\beta_x^{(i)}$, $\kappa_t^{(i)}$, and γ_{t-x} for each factor *i*, age *x*, year *t* and cohort $c = t - x$ by maximizing the log-likelihood function.

4.2. Model Fitness

The experiments used the BIC to measure the performance of the seven stochastic models discussed in section 2. Different results were obtained from each country's data.

Table 2 shows the ranking of the models using the data from Japan. The top three models are the CBD extension model (CBDE), CBD2 and CBD3 according to BIC. As shown in the top three models for the UK are RH, CBDE and CBD2 according to BIC. All of the cohort extensions of CBD, but not the original CBD model, showed the advantages of a longevity risk model. The CBD variants CBD2 and CBD3 (Cairns and David et. al., 2009) contained the cohort component and produced excellent results in all countries. However, the original two-factor model CBD0 [5] produced poor goodness-of-fit in all countries. These results imply that models that adjust for the cohort effect can model realistic mortality better than can the original CBD model. However, the CBDE model was modified from CBD2, which added age-specific coefficients to capture different age effects. The results show that the proposed extension of the CBD model (CBDE) is promising and that it obtained the best BIC using the data from Japan and ranked second using the data from the UK.

Table 2: The LLF and BIC of Mortality Data of Japan

Model	Effective number of parameters	Maximum log-Likelihood	BIC	Rank
LC	94	-22815	-23143	6
RH	187	-7124	-7776	4
CBD0	72	-27207	-27457	7
CBD1	135	-8056	-8527	5
CBD2	**170**	**-6881**	**-7473**	**2**
CBD3	**137**	**-7042**	**-7519**	**3**
CBDE	**200**	**-6712**	**-7314**	**1**

Table 3: The LLF and BIC of Mortality Data of the UK

Model	Effective number of parameters	Maximum log-Likelihood	BIC	Rank
LC	94	-11571	-11898	7
RH	187	-7285	-7536	5
CBD0	72	-10207	-10458	6
CBD1	**135**	**-7143**	**-7622**	**3**

Table 3: The LLF and BIC of Mortality Data of the UK - continued

CBD2	**170**	**-6768**	**-7464**	**1**
CBD3	137	-7153	-7630	4
CBDE	**200**	**-6872**	**-7525**	**2**

Figure 1: Japan

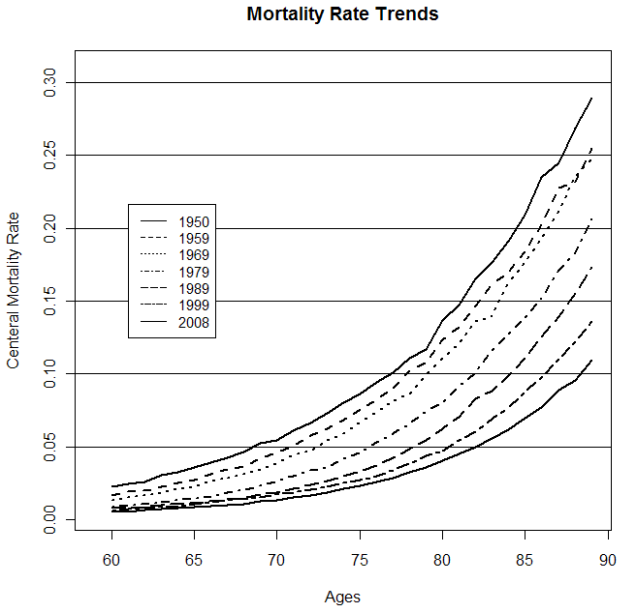
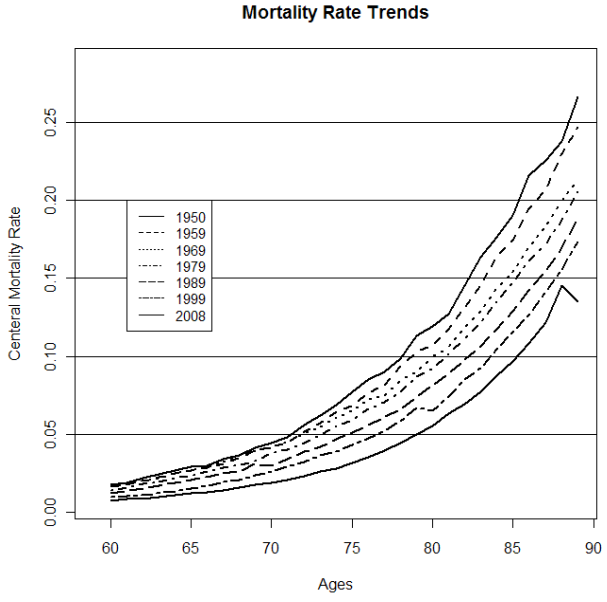


Figure 2: UK



4.3. Robustness of the Stochastic Models

To evaluate the robustness of the models, their parameters were re-estimated with the maximum likelihood method using the time period 1980-2008. The parameter trends of the seven mortality models for UK and Japan were plotted. The robustness analysis of the models is presented in the appendix (from Figure 3 to Figure 16).

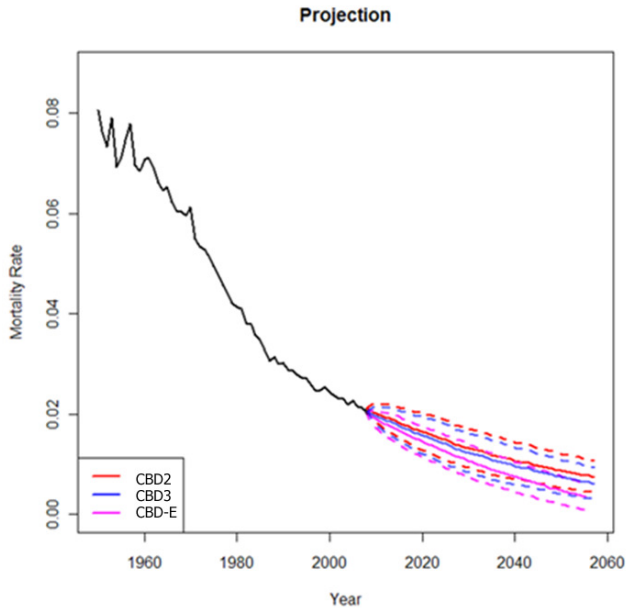
Cairns, David et al. (Cairns and David et. al., 2009) provided an excellent approach to the evaluation of the robustness of mortality models, suggesting that a robust model should have parameters that are consistent with different fitting periods. The figures in the appendix show the parameter trends of the evaluated models with distinct fitting periods from Japan. For example, Figure 4 shows that the RH model has four estimated parameters, but the model cannot exhibit consistent parameter trends for the two fitting periods. Of the seven models, only CBD0, CBD1 and CBDE model show stable trends between the two periods (Figure 5, Figure 6 and Figure 9). The other models either overestimate or underestimate behavior in 1968-2008 compared with 1980-2008. Although CBD0 shows fairly stable trends in Japan, the CBD2, CBD3 and CBDE models include a cohort component that could reflect more realistic mortality improvement than CBD0. However, CBDE has more stable trends than CBD2 and CBD3. Consequently, the CBDE model is a robust model for Japanese mortality data.

Similarly, for the mortality data of UK, the CBD0, CBD1 and CBDE models also show stable trends between the two periods (Figure 12, Figure 13 and Figure 16). In addition, LC and CBD2 models also provide steady trends. Consequently, according to BIC and parameter robustness, both CBD2 and CBDE models are robust model for mortality data of the UK.

4.4. Future Mortality Rate Projections

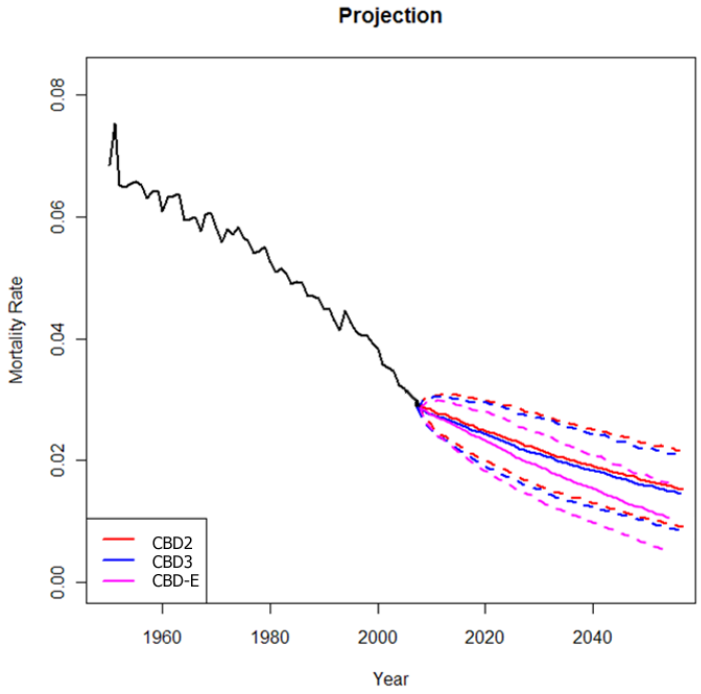
Using the results of the mortality models from the previous section to project future mortality rates is interesting and informative. We used the top three models identified in the BIC analysis to project the mortality rate at age 75 in 2009-2057[3]. Figure 17 shows the projection results from Japan with the CBD2, CBD3 and CBDE models. These models are illustrated by different colors with 95% confidence intervals (the median line is the best estimate). The projection from CBD3 almost overlays that from CBD2 but exhibits a slightly declining trend when compared with CBD2. The projection from CBDE (magenta lines) shows a greater decline in the mortality rate than CBD2 (red line) and CBD3 (blue line). Figure 18 plots the mortality rate projections from the UK. The results from the UK are similar to those from Japan but include a wider confidence interval; CBDE again shows a greater decline than CBD2 and CBD3.

Figure 17: Projected mortality rates at age 75 in future periods (2009 to 2057) in Japan via CBD2, CBD3 and CBD-E



[3] The projections for other ages are available from the author.

Figure 18: Projected mortality rates at age 75 in future periods (2009 to 2057) in the UK via CBD2, CBD3 and CBD-E



5. Conclusion

Longevity risk, or a lower than expected human mortality rate, is a crucial issue affecting governmental policies and the insurance industry. The Lee-Carter (LC) model (Lee and Carter, 1992) is perhaps the most popular approach to mortality modeling; however, various disadvantages arise in connection with the Lee-Carter model. One improvement is to consider the cohort effect in the classical LC model. These include Age-Period-Cohort (APC) models such as the Renshaw and Haberman (RH) model(Renshaw and Haberman, 2006) and CBD models (Cairns and Blake et. al., 2006b; Cairns and David et. al., 2009). APC models attempt to add a cohort effect, and CBD models aim to model risk among older age groups.

To create a more suitable model and address longevity risk, this study proposes an extension of the CBD model (CBDE). The CBDE model is inspired by the CBD2 model, which integrated cohort effects and added age-specific coefficients to account for different age effects among various age groups. This study evaluated the models for goodness-of-fit and robustness. Section 4 showed that CBDE resulted in the best goodness-of-fit and exhibited better robustness than the other models. In particular, CBDE showed stable parameter trends between short-term and long-term periods. Although CBD-E did not produce the best BIC values with data from all countries, this model showed the greatest robustness. These results suggest that CBDE may allow for more stable mortality rate projections.

Appendix

Figure 3: Japan data: Parameter estimates for the LC model

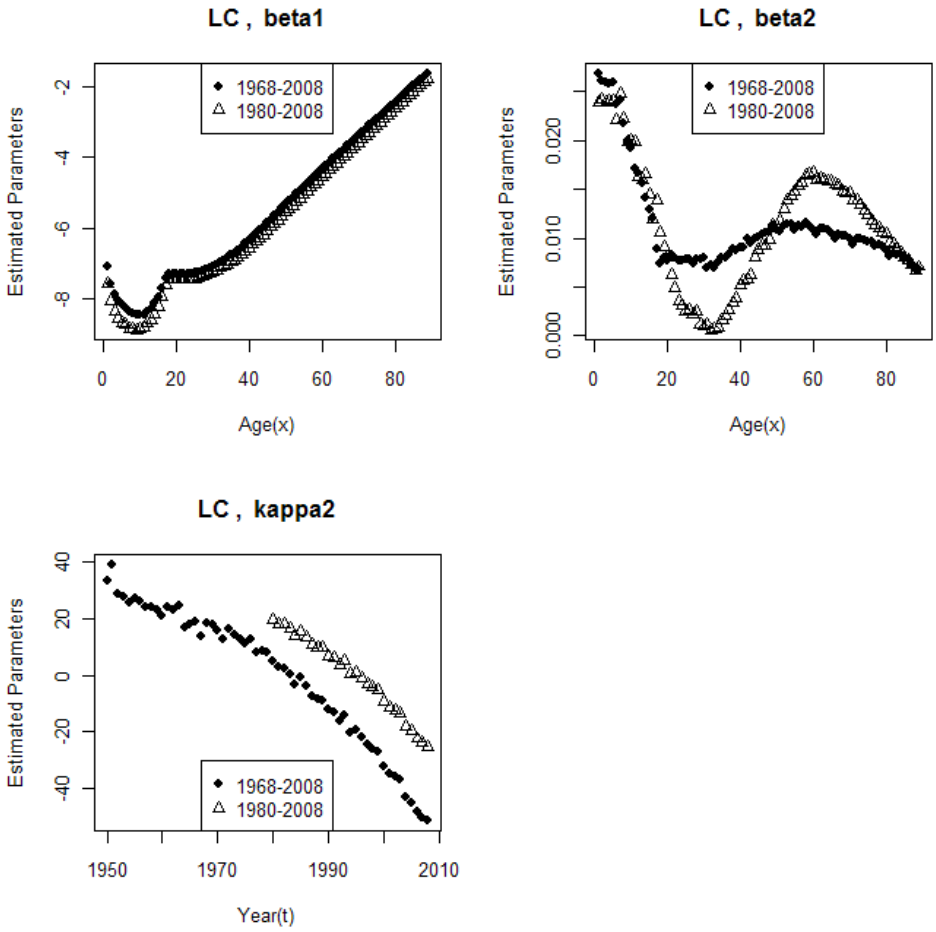


Figure 4: Japan data: Parameter estimates for the RH model

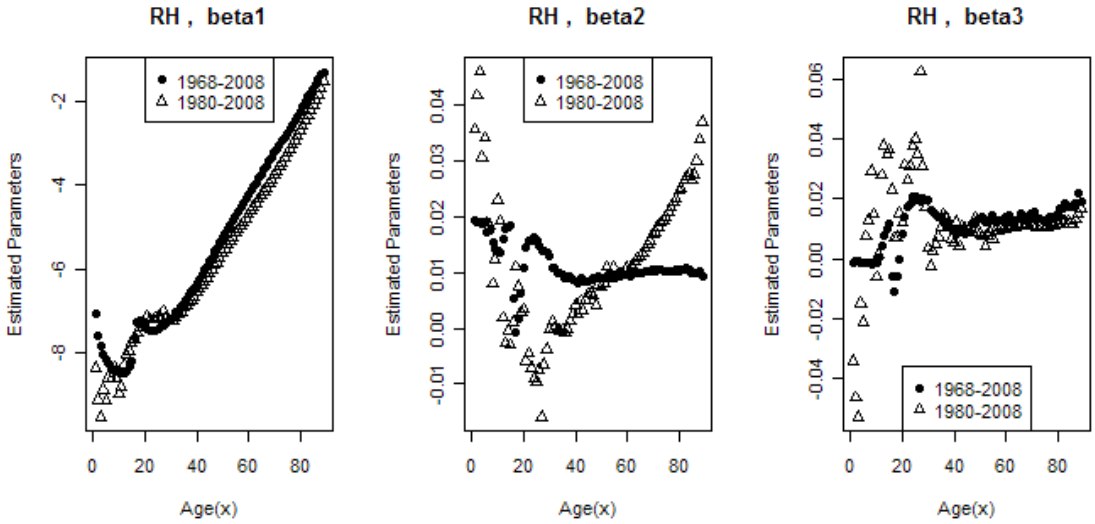


Figure 4: Japan data: Parameter estimates for the RH model - continued

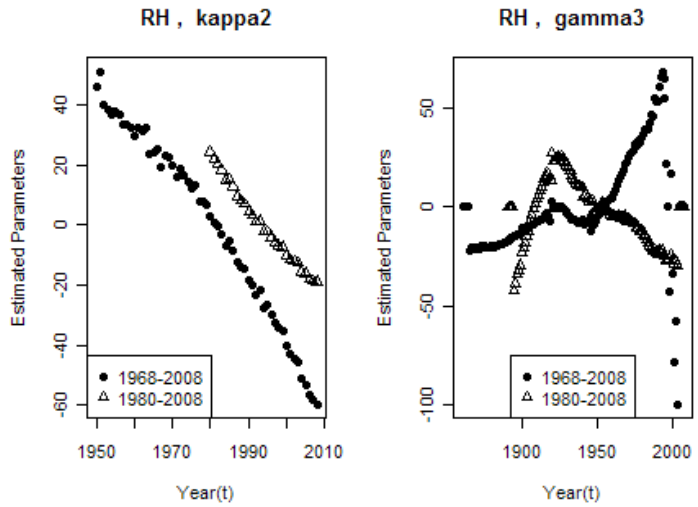


Figure 5: Japan data: Parameter estimates for the CBD0 model

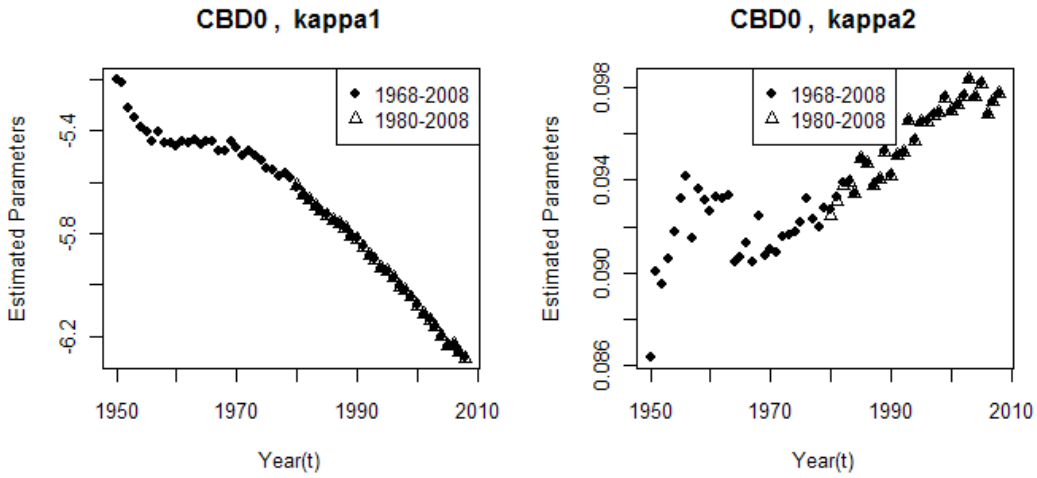


Figure 6: Japan data: Parameter estimates for the CBD1 model

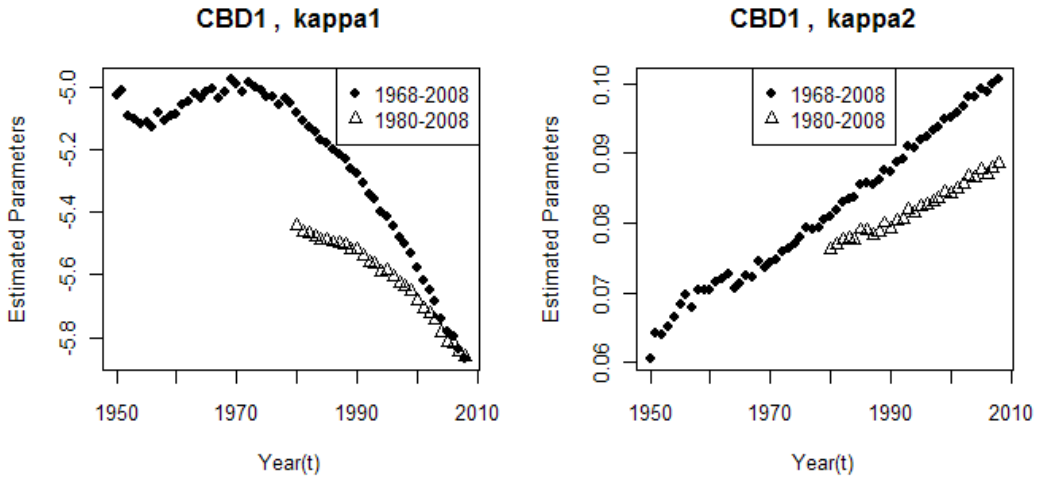


Figure 6: Japan data: Parameter estimates for the CBD1 model - continued

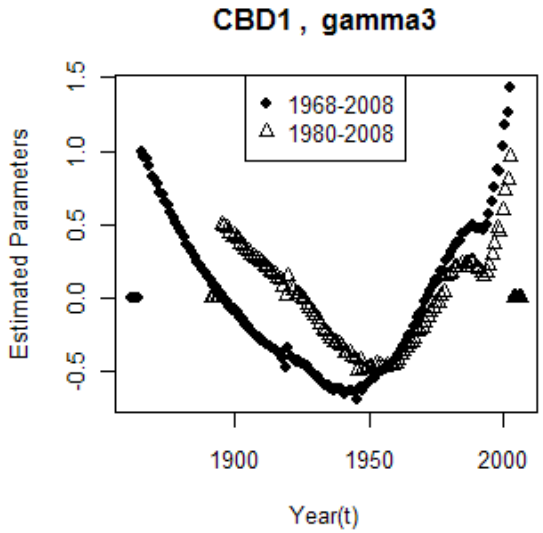


Figure 7: Japan data: Parameter estimates for the CBD2 model

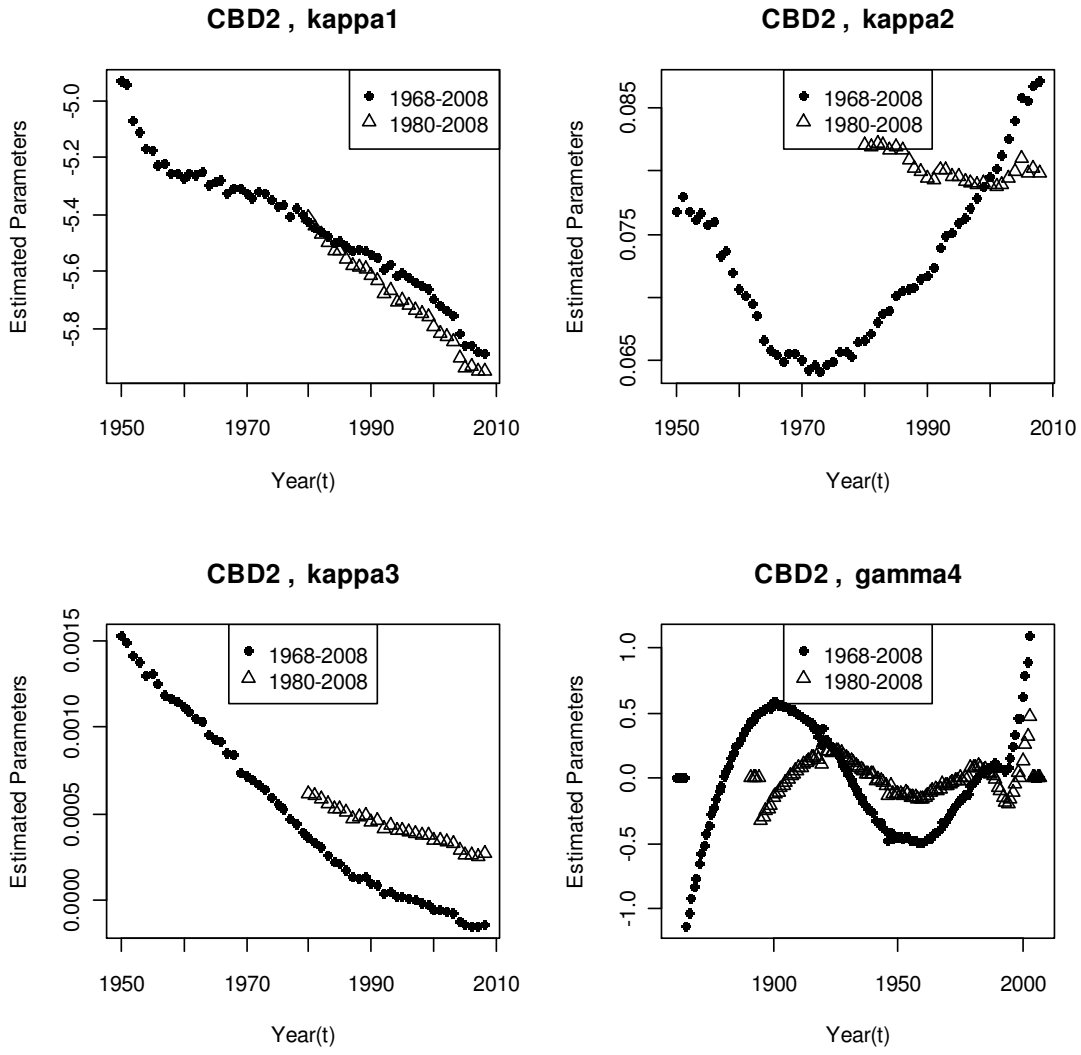


Figure 8: Japan data: Parameter estimates for the CBD3 model

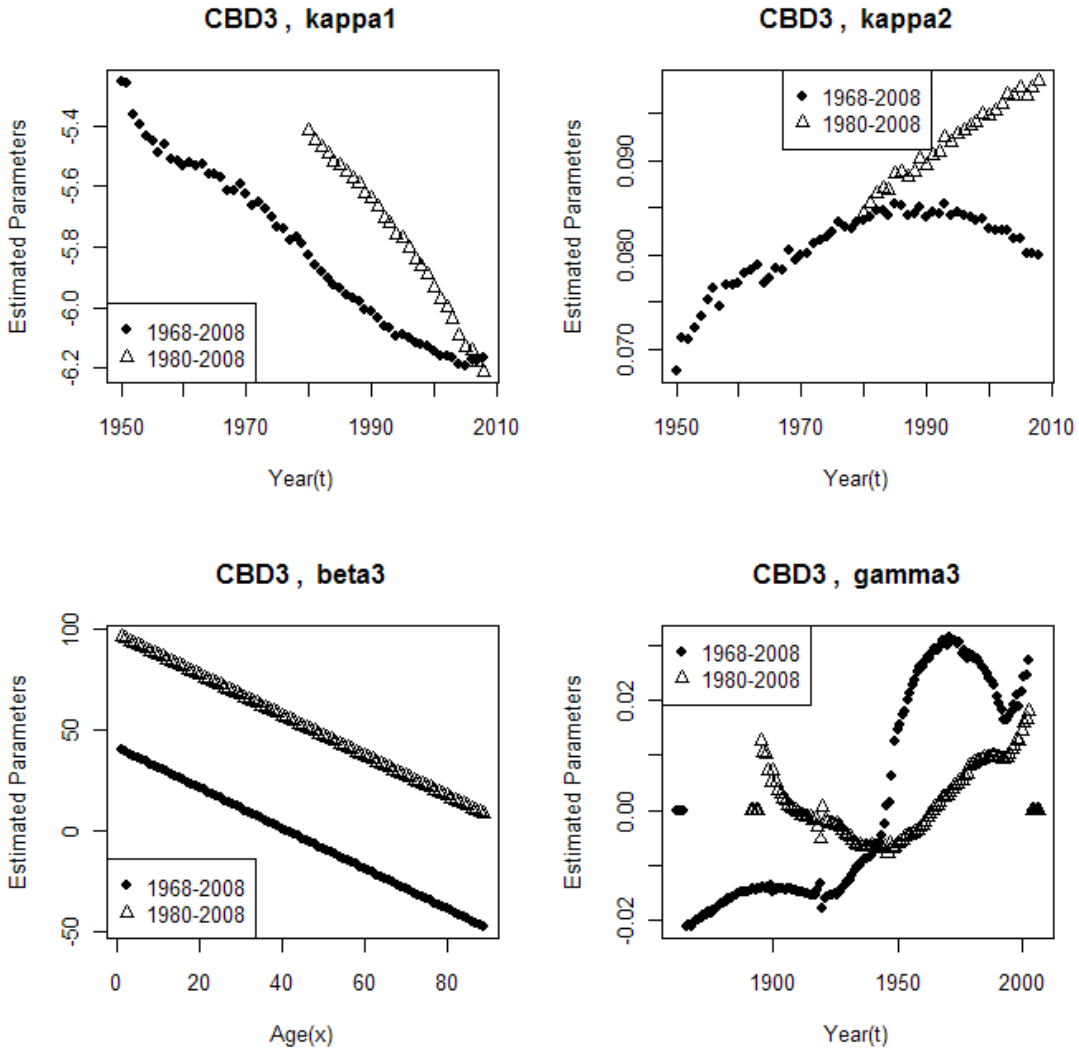


Figure 9: Japan data: Parameter estimates for the CBD-E model

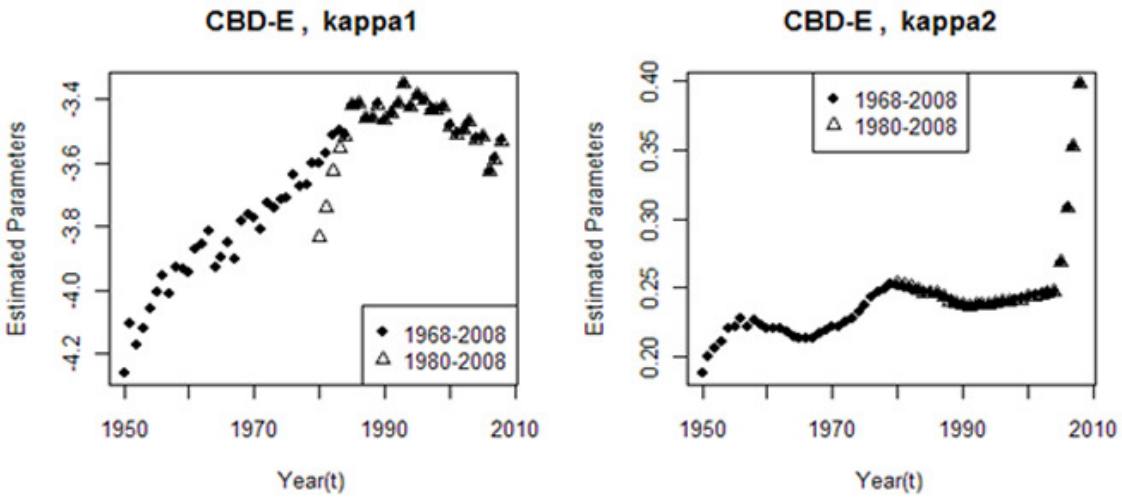


Figure 9: Japan data: Parameter estimates for the CBD-E model - continued

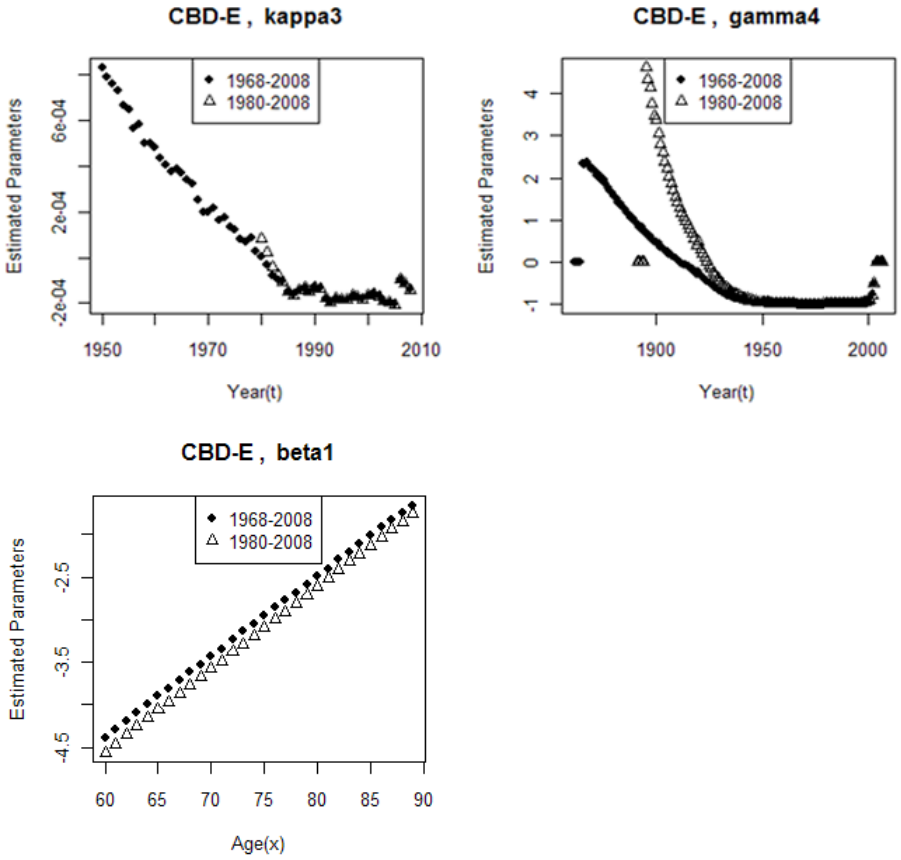


Figure 10: UK data: Parameter estimates for the LC model

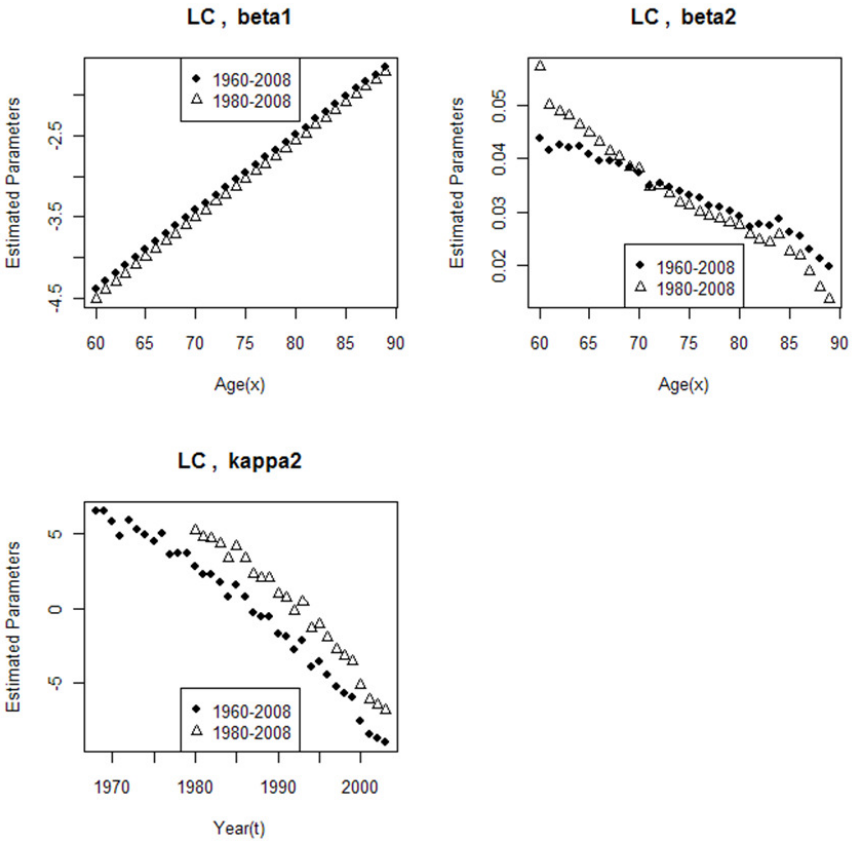


Figure 11: UK data: Parameter estimates for the RH model

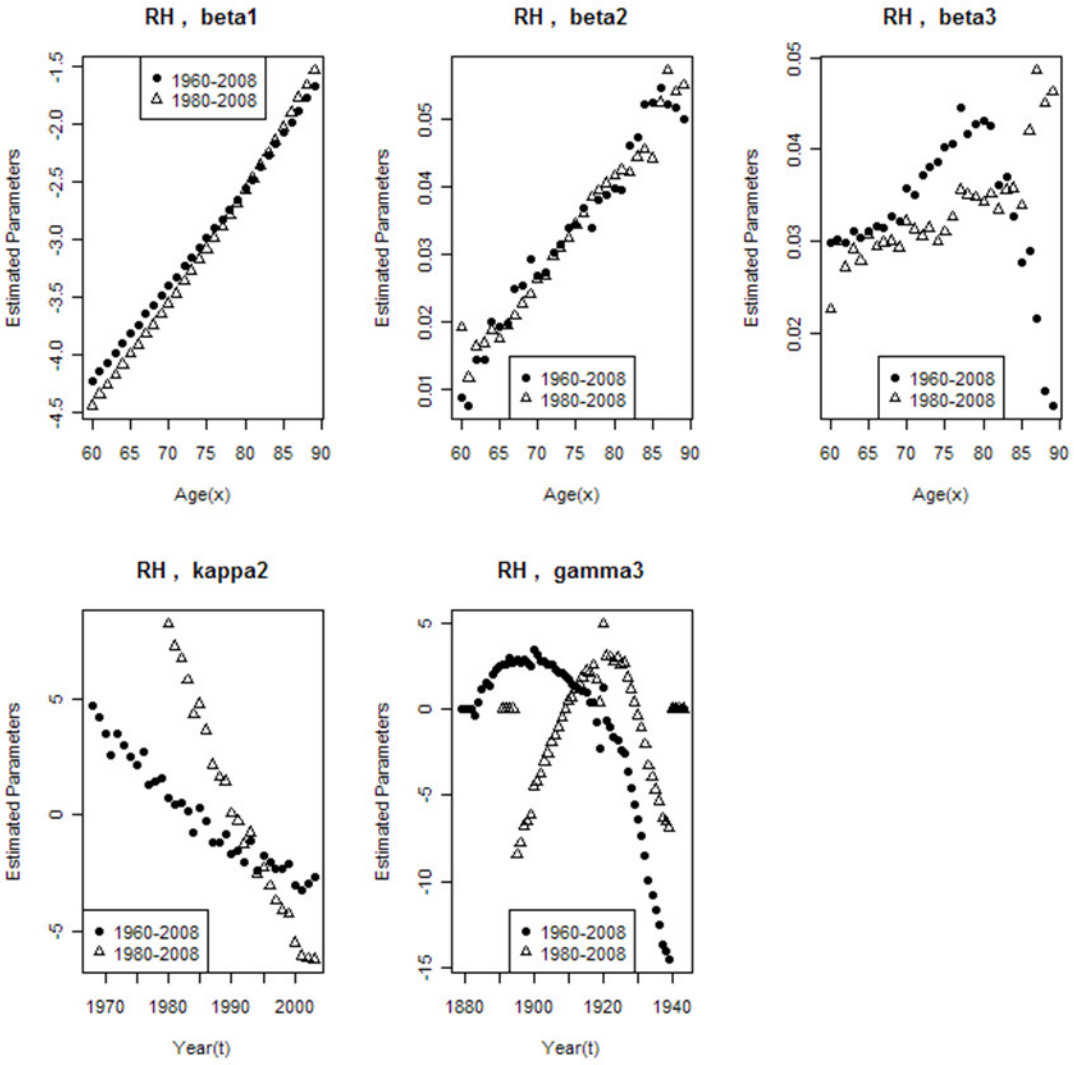


Figure 12: UK data: Parameter estimates for the CBD0 model

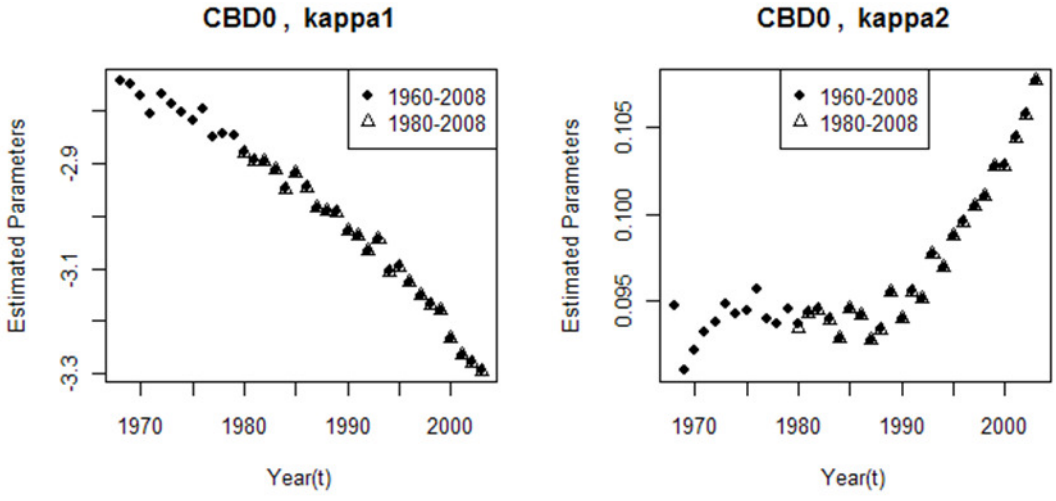


Figure 13: UK data: Parameter estimates for the CBD1 model

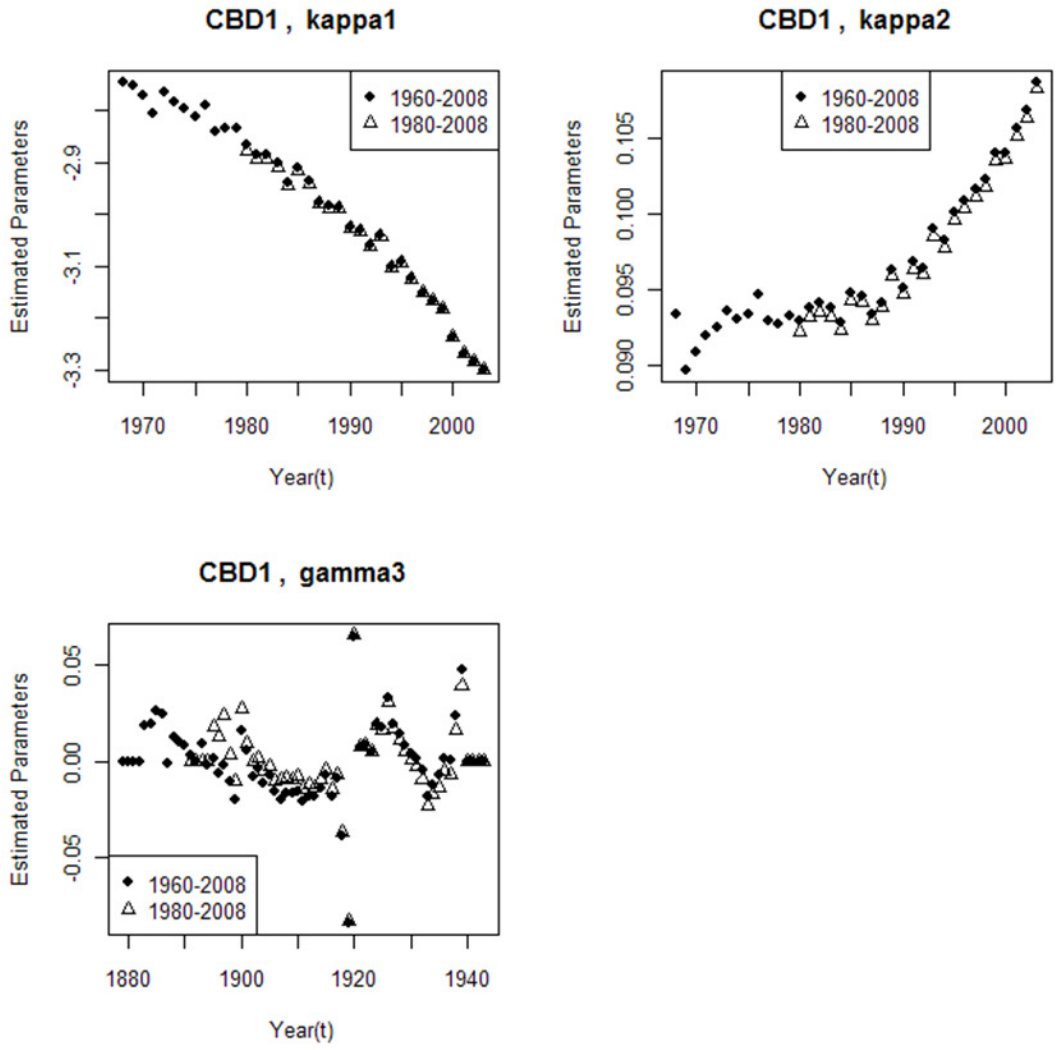


Figure 14: UK data: Parameter estimates for the CBD2 model

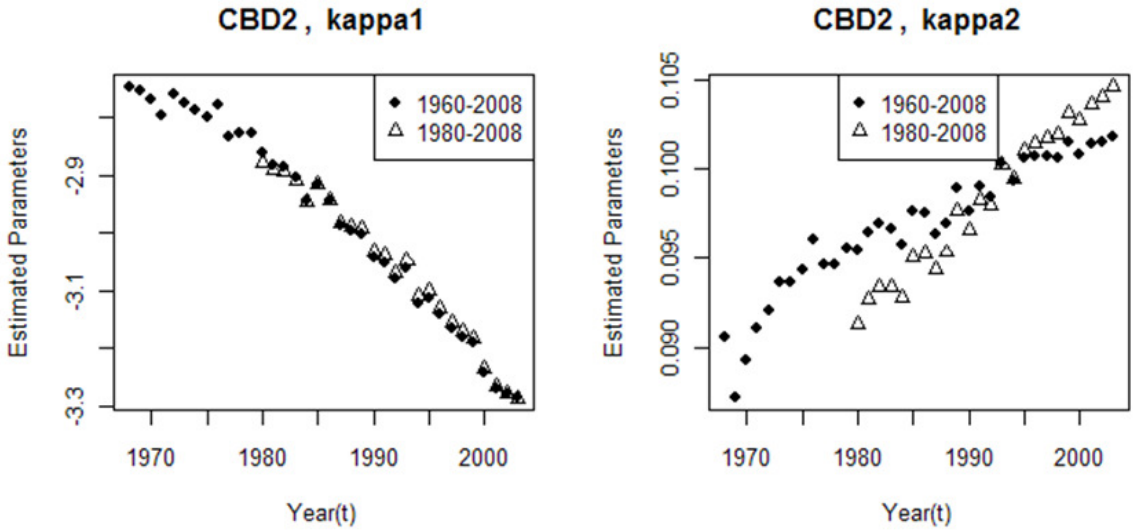


Figure 14: UK data: Parameter estimates for the CBD2 model - continued

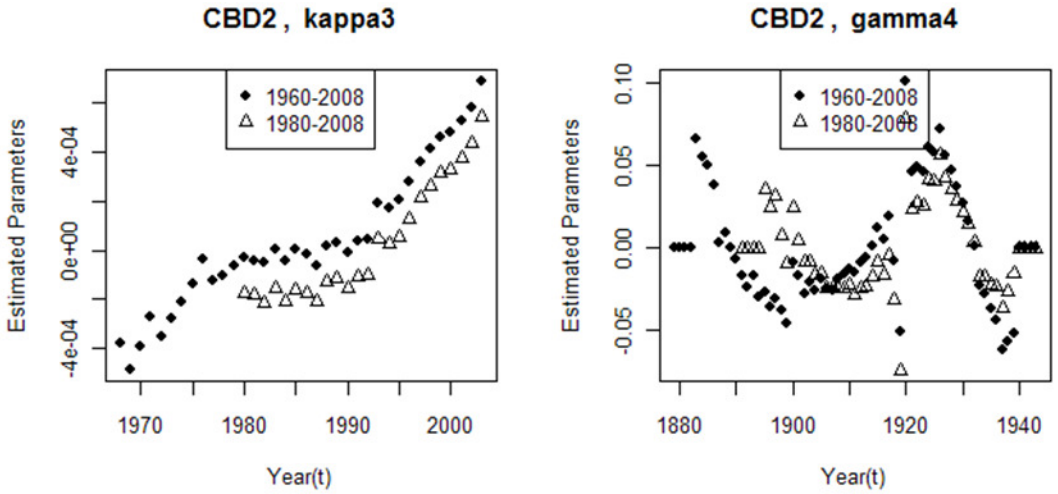


Figure 15: UK data: Parameter estimates for the CBD3 model

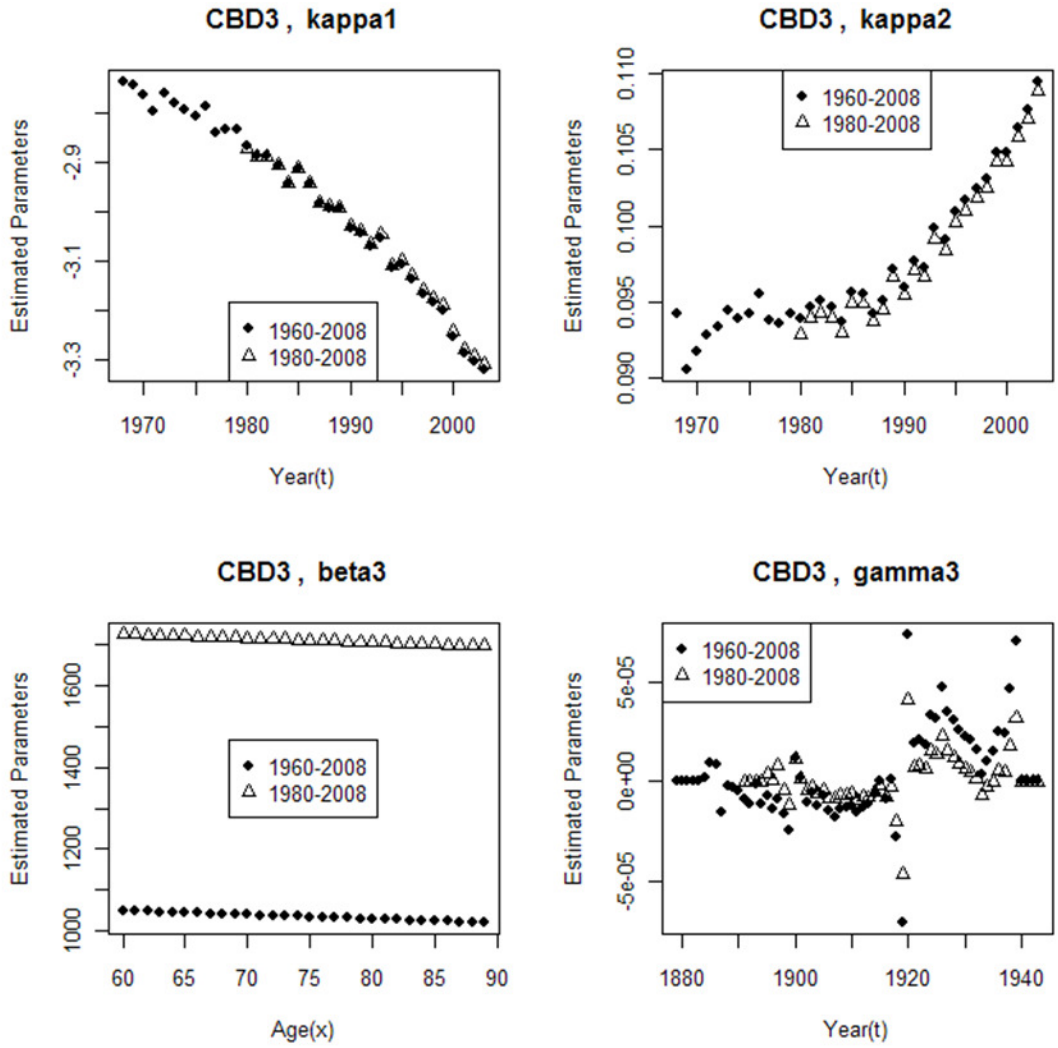
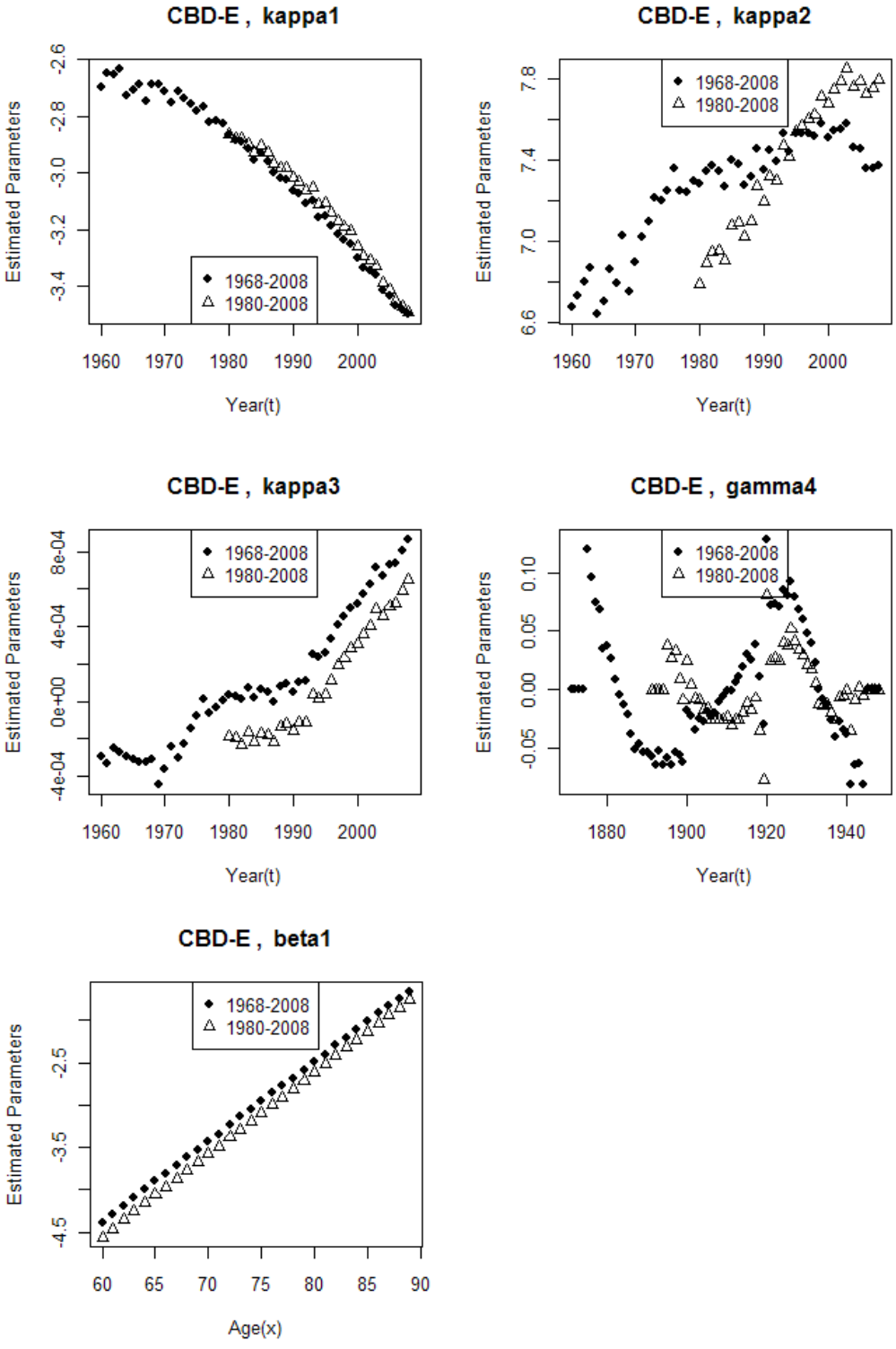


Figure 16: UK data: Parameter estimates for the CBD-E model



References

[1] Biffis, E. (2005). "Affine processes for dynamic mortality and actuarial valuations." Insurance: Mathematics and Economics **37**(3): 443-468.

[2] Blake, D., A. J. G. Cairns, et al. (2006). "Living with mortality: Longevity bonds and other mortality-linked securities." British Actuarial Journal, in Press.

[3] Cairns, A. J. G. (2007). "A multifactor generalisation of the Olivier-Smith model for stochastic mortality." Proceedings of the 1st IAA Life Colloquium.

[4] Cairns, A. J. G., D. Blake, et al. (2006a). "Pricing death: Frameworks for the valuation and securitization of mortality risk." ASTIN Bulletin **36**(1): 79-120.

[5] Cairns, A. J. G., D. Blake, et al. (2006b). "A two-factor model for stochastic mortality with parameter uncertainty: Theory and calibration." Journal of Risk and Insurance **73**(4): 687-718.

[6] Cairns, A. J. G., D. Blake, et al. (2008). "Mortality density forecasts: An analysis of six stochastic mortality models." Working paper, Heriot-Watt University, and Pensions Institute Discussion Paper PI-0801.

[7] Cairns, A. J. G., B. David, et al. (2009). "A quantitative comparison of stochastic mortality models using data from England and wales and the United States." North American Actuarial Journal **13**(1): 1-35.

[8] Chen, H. and J. D. Cummins (2010). "Longevity bond premiums: The extreme value approach and risk cubic pricing." Insurance: Mathematics and Economics **46**(1): 150-161.

[9] Coughlan, G., D. Epstein, et al. (2007) "Lifemetrics Technical Document."

[10] Cox, S. H., Y. Lin, et al. (2010). "Mortality risk modeling: Applications to insurance securitization." Insurance: Mathematics and Economics **46**(1): 242-253.

[11] Dahl, M. (2004). "Stochastic mortality in life insurance: Market reserves and mortality-linked insurance contracts." Insurance: Mathematics and Economics **35**(1): 113-136.

[12] Dahl, M. and T. Møller (2006). "Valuation and hedging of life insurance liabilities with systematic mortality risk." Insurance: Mathematics and Economics **39**(2): 193-217.

[13] Lee, R. D. (2000). "The Lee-Carter method for forecasting mortality, with various extensions and applications." North American Actuarial Journal **4**(1): 80-93.

[14] Lee, R. D. and L. R. Carter (1992). "Modeling and forecasting U.S. mortality." Journal of the American Statistical Association **87**(419): 659-671.

[15] Milevsky, M. A. and S. David Promislow (2001). "Mortality derivatives and the option to annuitise." Insurance: Mathematics and Economics **29**(3): 299-318.

[16] R Development Core Team (2010). R: A Language and Environment for Statistical Computing. Vienna, Austria.

[17] Renshaw, A. E. and S. Haberman (2006). "A cohort-based extension to the Lee-Carter model for mortality reduction factors." Insurance: Mathematics and Economics **38**(3): 556-570.

[18] Schrager, D. F. (2006). "Affine stochastic mortality." Insurance: Mathematics and Economics **38**(1): 81-97.

[19] Wills, S. and M. Sherris (2010). "Securitization, structuring and pricing of longevity risk." Insurance: Mathematics and Economics **46**(1): 173-185.

[20] Wilmoth, J. R. (1996). "Mortality projections for Japan: A comparison of four methods." Health and Mortality Among Elderly Populations: 266-287.